Perceptive Design and the Artificial Inner Eye

Zippora Arzi–Gonczarowski
Typographics Ltd, Jerusalem 96222, Israel

1999 draft

Abstract

This paper summarizes a study that formalizes and analyzes artificial perceptual transitions from perceptions of actual environments to an internal conception of designed environments that are based on possible modifications of the actual environment. Such transitions are typically necessary when an artificial intelligent artifact needs to devise a plan for a designed manipulation of its environment. The formalization is performed within a mathematical framework that has been used before to formalize other artificial perceptual cognitive processes. The mathematical infrastructure consists of a category of 'artificial perceptions'. Each 'perception' consists of a set of 'world elements', a set of 'connotations', and a three valued (true, false, undefined) predicative connection between the two sets. 'Perception morphisms' describe paths between perceptions. It is shown here how an internal conception of modified environments can be categorically formalized by natural transformations. The formalization extends and systematizes pre-theoretical intuitions about entities that are 'more conceivable' than others, about perceptual acuity that guides a resourceful construction of conceived options, and about related issues. The results of this study are then compared with existing formalizations of other cognitive processes within the same categorical premises. This highlights similarities and connecting threads between seemingly distinct cognitive processes, such as reasoning and design.

1 Introduction

Perceptive creativity is closely related to the complementarity of analysis and synthesis. A salient designing intelligence is often characterized by an acute analysis of the environment in which its creative processes take place. Its perception provides for a discernment of constituent elements that are meaningful for the effects of its design. The creative perception then conceives of a recomposition of such constituent elements together to provide, by means of a creative synthesis, a new design. Apprenticeship in many domains is, for example, based on this paradigm. Trainees follow a perceptive study of the works of their masters, replicate them, and in due time, having internalized the principles of the domain, they might become creative, conceiving and designing novel works of their own.

Suitably observed relations among the constituents play an important role in the conception of new designs, since there are often restrictions on the manner in which entities can be composed out of their constituent parts. However, even with an intensive and thorough observation of relations among constituent parts, 'pure' analysis, then synthesis, of relevant constituent parts seems not to be enough as a basis for a new effective design. 'The whole is more than the sum of its parts', so that there should be more to a creative design of a cohesive whole than merely a structured 'hanging together' of pointlike constituents. (This is, perhaps, due to further constituents that either elude, or are ungraspable by, the perceiving artifact.) In order for a perceptive design of a cohesive whole to emerge properly out of its constituent parts, a designing intelligence should invariably refer to the actual environment in which it operates. The phenomena in this environment represent actual cohesive wholes that should ground internal conception of designs and their prospective realizations.

A mathematical theory of artificial perceptions is proposed as a framework for the formalization of artificial creative design. The mathematical definition naturally provides infrastructure for artificial creative design because it is based on unmediated, primitive impressions and discernments of constituents of cohesive wholes in the environment of the perceiving artifact. Labeled primitives of perceptual sensory-motor-neural origin enable embodied observation and interaction with the environment, as well as a cognitive mental conception of design, rational evaluation, planning, and execution that are grounded in an actual environment.

The premises for the mathematical theory are provided by a basic mathematical category [5]. Its objects are *perceptions*, consisting of *world elements, connotations*, and a three-valued *true, false, undefined* predicative correspondence between them. Morphisms describe paths of flow between perceptions. The theory provides a proposal for a 'unified standard' for the description and analysis of perceptual cognitive processes. This infrastructure has already been applied to other perceptually based processes:

- [5] discusses transitions, comparisons, and joins of various perceptions of the same environment.
- [2] deals with with transitions between perceptions of analogical environments and with resulting metaphorical perceptions.
- [4] takes up productions of mental representations: meaningful cognitive images of the environment that are generated over the sensory-motor-neural apparatus.
- [3] goes into the coupling of sensations with emotive reactions, and studies the resulting, complex, affective–cognitive behavior.

Problems and conjectures can be meticulously stated and results unequivocally inferred and concluded using formal, context free, premises. Mathematical rigor is, indeed, essential in an artificial context. However, whenever a result is reached, it should be examined and compared to existing intuitions, opinions, and theories from other disciplines that deal with creative-imaginative perceptions and related cognitive processes.

The Mathematical categorization provides a setting for the approximation of intelligent creative perception by describing particular models of such perceptions as well as foundations for a general account of intelligent creative perception. [23] states that: 'Even within mathematical experience, only ... (category) ... theory has approximated a *particular* model of the general, sufficient as a foundation for a *general* account of all particulars'. Marrying mathematical category theory with cognitive studies is also proposed in [24], who suggest 'that universals of the mind may be expressed by means of universal properties in the theory of categories', and that categorical universals constitute 'blueprints' of the mind.

It is shown that the mathematical categorization can naturally formalize processes of artificial creative design which is a salient cognitive activity. The novelty of the approach consists of the following:

- The introduction of a mathematical categorical framework to formalize creative design, where no such framework already exists. The advantages of mathematical formalizations as analyzed, for example, in the introduction to [9] include clarity, precision, versatility, generalizability, testability, allowance to model complex phenomena that are far too complex to be grasped by a verbal description, and allowance to use results of a well developed science.

- Within the proposed mathematical categorical framework, the idea of perceptions of *conceived* environments is novel, as well as *Boolean environments* and *Boolean closures of environments.*

- A fallout of the mathematical categorization is an insight into 'connatural' cognitive processes. Whenever two cognitive processes or capabilities are both captured by similar mathematical constructs, this means that they are 'connatural': an artificial perception that can perform one of them should be able to perform the other. It turns out that formal constructions of this work repeat similar constructs related to cognitive processes other than design, and hence one also gains a novel insight into connaturality of cognitive capabilities and processes.

Human intelligence has inherently been a source of inspiration for research in AI, and this study is no exception. However, this work is not committed to being empirically adequate from a human psychological point of view.

2 The Mathematical Toolkit

For immediate and easy reference, the mathematical premises are repeated from the works that are cited in the introduction. The feature that various perceptually cognitive processes are repeatedly based on exactly the same premises warrants a unified theory, as solicited, for example, by [26], [27], and [1].

2.1 Perceptions as Mathematical Objects

The abstract idea of a perception is postulated as a mathematical construct which relates between phenomena outside the artificial agent, a set of *world elements*, and reflections which are internal to the artificial agent, a set of *connotations*. Every perception has its own set of w-elements, its own set of connotations, and its own predicative correspondence between the sets. Perceptions are high-level in the sense that they reside higher than pixels on the screen or waveforms of sound. They are object centered and therefore happen at and above the level of recognition of cohesive wholes, where meaning and conscious cognizance begin to play a role.

Definition 1 *A* **Perception Machine** *(* **Perception** *for short) is a three-tuple,*
$\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$*, where:*

- $\mathcal{E}$ *and* $\mathcal{I}$ *are finite, disjoint sets.*
- ϱ *is a 3-valued predicate* $\varrho : \mathcal{E} \times \mathcal{I} \to \{t, f, u\}$

The set $\mathcal{E}$ represents the outside, objective, world which the machine perceives. Anything which exists independent of the artificial perception, and could perhaps be discerned by it, could be a legitimate element of $\mathcal{E}$, and hence a w-element. Example w-elements may be a sounding bell, a light bulb, a blow of wind, smoke from a chimny (smelly or not), a candy bar, etc. These example w-elements are typically discerned by the human sensory-motor-neural apparatus, but some artificial perceptions may be unable to discern them. Artificial perceptions may, however, discern w-elements that are imperceptible for humans, such as certain kinds of radiation. Furthermore, different perceptions might break the same reality into different chunks. For example, wherever one perceives a single cohesive w-element 'box', another may perceive an arrangement of six w-elements 'board'. For humans, a human face would usually be a single, cohesive, w-element that is easily perceived. Whether this is also the case where machine perception is involved, is, however, not so clear. Hence, although the external environment is assumed to have an objective existence, its division into w-elements depends on the specific perception. (This phenomenon, as related to humans, has been studied by gestalt psychology [10]).

The set $\mathcal{I}$ stands for the internal representation, the ontology, of w-elements. Its elements have a subjective existence dependent on the machine. Anything which may be stored and manipulated in the machine (words, symbols, icons, etc.) could be a legitimate element of $\mathcal{I}$, and hence a **connotation**. Example connotations may stand for the pitch and/or duration and/or timbre and/or volume of a sound, the brightness and/or hue and/or saturation of a light, etc. These example connotations typically represent attributes or properties that are measurable by humans, and hence considered 'objective'. However, 'hot' and/or 'dark' and/or 'good' and/or '?!?!?' are legitimate connotations as well (the last one is not a typo). These are definitely not 'objective', they depend on the specific perception.

The three-valued predicate ϱ is the **Perception Predicate** (p-predicate for short) which relates w-elements and connotations, providing the connection between the environment and its internal representation. The terminology for the various ϱ values will be the following:

- $\varrho(w, \alpha) = t$, it will be said that w *has connotation* α.
- $\varrho(w, \alpha) = f$, it will be said that w *lacks connotation* α.
- $\varrho(w, \alpha) = u$, it will be said that w *may either have or lack this connotation*, the 'or' is evidently exclusive. This undefined value might eventually become defined but right now it is not.

Subsymbolic, early, sensory-motor-neural processing is innate to the agent and its architecture. Connectionism has been suggested as a candidate for the mechanism that underlies this lower level apparatus [19]. The emergence of higher level perception depends on predetermined, hard wired, sensory-motor-neural capabilities, and also on more fluent capabilities: on the agent‘s function and internal organization, on its gestalt perception, mental imagery, affective state, the situation at hand, etc. All these factors are supposed to be encapsulated in the definition of perception, and ϱ in particular.

Connotations that are alphabetic strings do not necessarily follow their dictionary definitions (if they have any). A smelly invisible vapor may, for instance, have the connotation 'pink'. This may depend on the agent's own individual architecture, programming and experience. Likewise, the issue of *why* the p-predicate has any one of the three values at a certain point simply warrants no discussion. As an example, the undefined u value of the p-predicate may be due to ignorance, irrelevance, future contingence or other reasons. From the philosophical point of view, these possible reasons are quite different one from the other. In the present context, however, the actual reason for a specific u value, or whether or not it is already 'decided' in some transcendental way, is irrelevant. Basic, individual, sensations and impressions need not be explained.

To illustrate the ideas of this study, quite a few examples will be based on perceptions of animals. The study, however, is context free, and no domain specific knowledge is required. Based on millennia of interaction with the animal kingdom, humans have been apprehending living creatures from namy perspectives. Cognitive analysis did not stop at mere classifications. Creative design and synthesis ensued when creatures were conceived for mythological or artistic purposes, for the scientific conception of extinct species, for purposes of hybridization of new species, and creativity in this domain continues with genetic engineering. Cognitive perceptions of animals could therefor provide examples of the complementarity of analysis and synthesis, as well as other related issues.

Consider a zoological garden environment where animals are the w-elements. Intelligent agents who might enter the zoo could feature different perceptions of this environment, varying with their sensory-motor-neural apparatus, scientific background, purposes for entering the zoo (enjoyment, academic), their attention to details, affective mood, etc.

Example 1 *Let $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$ be a 'tale' perception where:*
$\mathcal{I}$ = { *big, small, tail, wings, trunk, hooves, hump, fur, feather, udders, horns, ivory, crown, mane, beak, claws, striped, spotted, white, brown, colorful ... jumpy, sleepy, crawls, gallops, climbs, ... lives-in-water, lives-in-paddock, nocturnal, ... scary, angry, friendly, happy, sad, cuddly, ugly, beautiful, ...* }
For all animals w *in* $\mathcal{E}$ *and for all connotations* $\alpha \in \mathcal{I}$, $\varrho(\mathrm{w}, \alpha) = t$ *if and only if* w *has that connotation by* $\mathcal{P}$. *It is* f *if and only if* w *does not have that connotation by* $\mathcal{P}$. *It is* u *if* $\mathcal{P}$ *does not offer any perception of that connotation. In this example (and in other typical cases as well) connotations can be subdivided into 'families', such as body-part connotations, behavior connotations, habitat connotations, personal impression connotations, etc. However, they all share the same status as elements of* $\mathcal{I}$. *Gärdenfors [14, 15, 16] offers to make a distinction among different 'quality dimensions' that make a 'conceptual space'. A distinction in this spirit follows from Boolean constructions in [4].*

Classification of sense impressions of living organisms can be performed in many other, radically different, ways. It can be informal and superficial as in everyday use of words such as 'worm' or 'fish', or it can be based on various scientific ontologies such as 20th century systematics which is based on 'Darwinian' perception of evolutionary history, Linnaeus 'Systema Naturae' which is based on perception of idealized morphological traits, or Aristotle's 'Scale of Nature' which is based on a hierarchical perception of the animate world, and there are other approaches as well. One could even conceive of a variety of 'gastronomic' perceptions of fauna and flora with connotations pertaining to edibility and taste, or 'hunter' perceptions that classifiy game as prey according to methods, means and ease of pursuit and capture.

In apprehending the environment, mental processes typically presuppose classification of sense impressions, and design is, indeed, a salient mental process. The variety of connotations that an agent is at all able to perceive in the w-elements of its environment typically determines how divergent or creative it could be expected to be. Sometimes the novelty and originality of a design depend on the perception of a detail, a connotation of a w-element that has not been perceived before. Accordingly, the mathematical premises have just been based on a generalized, categorical, formalization of sense impressions. It is schematic in the sense that a possibly infinite number of perceptions is represented, with specific environments and perceptions as substitution instances.

2.2 Perception Morphisms

Intelligent apprehension of the environment, and creative processes in particular, often require flexibility of perception and an ability to step across boundaries. Environments are dynamic, and so are the ensued perceptions of these environments. Both environments and internal representations might change for a multitude of reasons. As a dynamic, 'fluid', phenomenon, perception needs channels for the flow of change. Also, given the variety and individuality of perceptions, one might need a way to bridge, if at all possible, between various perceptions. For example, passage from Aristotle's perception to modern perception of living organisms could be viewed as a transfer of the 'Scale of Nature', from the simplest to the most complex, to change in time, by evolution, from the simple to the complex. Linnaeus 'natural method of arrangement' can be viewed as an intuitive grasp of homologous relationships, reflecting evolutionary descent from a common ancestor. When different perceptions are somehow commensurate, then it is offered to categorize a generalized cognitive transition between them by *Perception Morphisms* (Morphisms are often nicknamed 'arrows' in the categorical literature).

Definition 2 *Let* $\mathcal{P}_1 = \langle \mathcal{E}_1, \mathcal{I}_1, \varrho_1 \rangle$ *and* $\mathcal{P}_2 = \langle \mathcal{E}_1, \mathcal{I}_2, \varrho_2 \rangle$ *be two perceptions.* $h : \mathcal{P}_1 \to \mathcal{P}_2$ *is a* Perception Morphism *(*p-morphism *for short) if* h *defines the following set mappings with a structure preserving condition:*

- $h : \mathcal{E}_1 \to \mathcal{E}_2$,
- $h : \mathcal{I}_1 \to \mathcal{I}_2$,
- No–Blur *is the structure preservation condition: For all* $\mathrm{w} \in \mathcal{E}_1$, *and for all* $\alpha \in \mathcal{I}_1$, *whenever* $\varrho_1(\mathrm{w}, \alpha) \neq \mathrm{u}$ *then* $\varrho_2(h(\mathrm{w}), h(\alpha)) = \varrho_1(\mathrm{w}, \alpha)$.

The definition provides possibilities to formally capture (combinations of) basic types of dynamic perceptual change:

- Addition of constituents (either connotations or w-elements) by mapping into larger sets of constituents.
- Change in the 'chunking' of constituents by many-to-one mappings.
- Unblurring of perception from undefined to defined values as specified by the No-blur condition.

A formalization of the reversed forms of these changes (namely: restriction of sets of constituents, or finer chunking of constituents, or blurring perception from defined to undefined values) will be offered in a short while, as a result of the categorical construction.

The fabrication of adequate mappings between different settings is, itself, a highly creative perceptual process, and computational models are documented in [25], [13], [20]. The view that mappings play a central role in cognitive processes is not new, and is shared, among others, with [12] and with research in analogy making, [11], [21], [29] to name just a few. The present approach goes a step further in proposing to use a context free, rigorous, mathematical infrastructure, and to scrutinize its implications by employing deductive mathematical tools. It will be shown that this leads to meaningful insights, as well as to a clear modeling of complex cognitive phenomena. The works that are cited in the introduction apply various types and combinations of p-morphisms to formalize the artificial cognitive processes they deal with. Formalization of design processes will be proposed below in terms of specially trimmed p-morphisms between perceptions. The mathematical properties of the p-morphism (isomorphism, one to one, many to one, onto, impossible etc.) carry meticulous information about how commensurate the involved perceptions are, and pinpoints differences between them.

2.3 A Word about Mathematical Formalizations

Definitions 1 and 2 are minimalistic. It is in the very nature of mathematical premises to be minimalistic. This is not necessarily synonymous with superficiality or over simplification. As an example, arithmetic is based just on numbers and four simple operations: addition, subtraction, multiplication, and division (there is actually some mathematical redundancy already in these premises for arithmetic). Further arithmetic objects are constructed in an orderly manner, and theorems express properties of the system of arithmetic objects that are assured by the mathematical mode of generation. Deductive reasoning is used to obtain unobvious arithmetic properties from obvious or immediate ones. (A selection of immediate properties which constitute the five postulates of number theory is due to Giuseppe Peano.) Orderly extensions of the natural numbers provide the integers, then the rational numbers, then the real numbers. Complex arithmetic computations in economy, technology, and science, which are neither superficial nor simplistic, are based on the arithmetic premises. The originator of this type of formal method, which dominates all areas of 20th century mathematics, and not only number theory and arithmetic, was David Hilbert.

The proposal to formalize artificial perceptions and related cognitive processes is an attempt to follow the beaten tracks of 20th century mathematical method. Based on the trim definitions of this section, further constructions and processes will be generated in a mathematical orderly manner, and theorems will capture properties of perceptions and perceptual transitions that will be assured by the mathematical mode of generation. Deductive reasoning will be used to obtain insights and complex properties from the basic ones. The formal scrutiny is a means rather than an end by itself. It will be justified later by showing that the mathematical categorical tools are useful and meaningful to the study of perceptual cognitive artificial processes, and design processes in particular.

2.4 Two sides of Artificial Perceptions

One salient property of definition 1 is the symmetry between $\mathcal{E}$, the environment, and $\mathcal{I}$, the representation. From a purely technical, context free, point of view, the roles that a w-element and a connotation play in the definition of the p-predicate are interchangeable. This *duality* has the following consequences:

- Technically, any formal construction or theorem that is established for connotations (w-elements) can automatically be applied to w-elements (connotations), mutatis mutandis.
- Since the proposed formal constructions and theorems capture cognitive processes, the technical symmetry could provide insights into similarities of cognitive processes and capabilities that could, in turn, entail AI architectural and applicational similarities.

In [2] the duality was employed to parallel the $\mathcal{I}$ mapping ($h : \mathcal{I}_1 \to \mathcal{I}_2$) of a p-morphism as the *interpretive* component of the transition, with the $\mathcal{E}$ mapping ($h : \mathcal{E}_1 \to \mathcal{E}_2$) of the same p-morphism, as the *literal-analogical* component of the transition. The $\mathcal{E}$ mapping is 'pro-synthetic' in that it takes cohesive, existing, w-elements as its basic building blocks and maps between them. The $\mathcal{I}$ mapping is 'pro-analytic' in that it 'breaks' impressions of cohesive whole into separate sense connotations as building blocks, and maps between them. Computationally, a mapping is a mapping, so that the schematic construct looks the same. The import for the programmer is that the same architectural modul could perform both transitions (with different parameters). From the artificial cognitive point of view these are mental capabilities that are 'connatural'.

Example 2 *Given the environment of animals, it has been suggested to map the connotations of Aristotle's 'Scale of Nature', from the simplest to the most complex, to connotations of change in time, by evolution, from the simple to the complex. This transition from Aristotle's perception of the animal kingdom to modern 'Darwinian' perception of the same living organisms is interpretive, 'pro-analytic' because it maps representations.*

Example 3 *Musical pieces are w-elements, they could be represented by various musical notation systems. A transition from the constituents of European pitch solmization (Do, Re, Mi ...) to modern alphabetical pitch solmization (A,B,C ...) of each musical piece, maps representations and is thus interpretive, 'pro-analytic'. (Example p-morphisms are provided in [5]). If, on the other hand,* *musical pieces remind the listener of other musical pieces whenever they feature similar harmony connotations and/or arouse similar feeling connotations, that mapping is literal-analogical, 'pro-synthetic'. It maps cohesive wholes.*

Example 4 *[2] describes a literal-analogical p-morphism from the camera environment to the eye environment that is based on a similarity of functional connotations. W-elements are mapped: the shutter is mapped to the iris, the film to the retina, etc. This transition is literal-analogical, 'pro-synthetic'. It maps environmental wholes.*

The structure preservation, *no-blur* condition, of definition 2 is crucial. It binds the interpretive and the literal-analogical aspects. It provides an analytic explanation to the mapping of w-elements on one hand, and, on the other hand, grounds the mapping of connotations in holistic experience.

The technical parallelism between interpretations (translations, communications) on one hand, and analogies on the other hand, provides a precise insight into the connaturality of these, seemingly different, processes. Not unexpectedly, design processes will be formalized as pro-synthetic, involving manipulation of cohesive wholes. They will be shown dual to processes that involve analytic, connotational representations.

2.5 Mathematical Categorical Context

(Readers not interested in mathematical context may skip this subsection.)

Having defined perceptions and perception morphisms, we define the *Category of Perceptions* as a basis for a mathematical theory of artificial perceptions. (In the same manner the infrastructure for group theory is provided by defining groups, group homomorphisms, and the category of groups.) All perceptions are regarded as a collection, and formalized as a mathematical category, $\mathcal{P}rc$. This provides infrastructure from a well developed mathematical domain: Category Theory. The definition of a category requires that:

- One is given a set of *objects*.
- Given any pair of objects $\mathcal{P}, \mathcal{P}'$, one has a collection of *morphisms* $f : \mathcal{P} \to \mathcal{P}'$ from $\mathcal{P}$ to $\mathcal{P}$'. Given a morphism such as f, $\mathcal{P}$ is the *domain* of f, and $\mathcal{P}$' is the *codomain* of f.
- Morphisms should be closed under composition: Given two morphisms $f : \mathcal{P} \to \mathcal{P}'$ and $g : \mathcal{P}' \to \mathcal{P}''$, where the codomain of f is the same as the domain of g, one may form their *composite*, $f \circ g$, which is a morphism: $f \circ g : \mathcal{P} \to \mathcal{P}''$, such that $f \circ g(a) = g(f(a))$ (i.e. apply f, then g).
- Composition should be associative: $f \circ g \circ h = (f \circ g) \circ h = f \circ (g \circ h)$.
- For every object $\mathcal{P}$ there should be an identity morphism $Id_{\mathcal{P}} : \mathcal{P} \to \mathcal{P}$.
- The identity morphism should be the (left and right) unit element of composition: For every $f : \mathcal{P} \to \mathcal{P}'$, $Id_{\mathcal{P}} \circ f = f = f \circ Id_{\mathcal{P}'}$.

In the present context the objects are perceptions $\mathcal{P}, \mathcal{P}' \ldots$ and morphisms are p-morphisms. Theorems and proofs are provided in [5], as well as a discussion of the basic mathematical properties of the category of perceptions, and some basic example applications.

In particular, a formalization of changes based on 'reversed p-morphisms' (namely restriction of the sets of constituents, or finer chunking of constituents, or blurring perception from defined to undefined values) as suggested after definition 2, is captured in $\mathcal{P}rc^{\mathrm{op}}$, the mirror category of $\mathcal{P}rc$. It has the same basic objects (namely perceptions), but instead of having arrows $\rightarrow$ which designate p-morphisms, it has arrows $\leftarrow$ which indicate reversed presentations of p-morphisms.

The construction and formalization of perceptual cognitive processes can be trimmed in terms of these very few primitives that category theory provides for the study of artificial perceptions: perception, p-morphism, domain perception and codomain perception of a p-morphism, and composition of p-morphisms. This predicts theoretical as well as applicational tidiness.

3 From Representations to Environments

In this section formalized cognitive processes are introduced that capture the intuitive idea that a perceptive artifact could specify and manoeuver subsets of connotations in order to give rise to a design.

The definitions started from perceptions of actual environments. W-elements in these environments are cohesive wholes that are susceptible to acute analysis by a perceiving intelligence. Natural primitive constituents for creative design are provided by the w-elements and the connotations that are discerned by the perceiving intelligence. Following an interanlization of the detailed impressions of the given, actual, environment, a creative perception may then proceed to conceive of a recomposition of constituent elements together to provide, by means of a creative synthesis, a new design.

The mathematical toolkit can be applied now to formalize the process. Given a perception $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$, a w-element w can be 'abstracted' as the set of connotations that it has: $\{\alpha \| \varrho(w, \alpha) = t\}$, and the set of connotations that it lacks: $\{\alpha \| \varrho(w, \alpha) = f\}$. These sets are typically idiosyncratic to the perception that is involved, as explained before. A formalization of a creative design of new w-elements and new environments could sometimes consist of a careful, perceptive, internal recomposition of such sets for w-elements that do not exist in $\mathcal{E}$.

Example 5 *Based on the set of connotations of the 'zoo tale' of example 1, and a recomposition of subsets of connotations, a perceptive intelligence could conceive of fantastic creatures just by specification of subsets like { big, tail, wings, horns, claws, black, nocturnal, scary, angry, ugly, ... } The basic perception that is involved determines what can and what cannot be conceived. A perception with color connotations, for instance, lays the foundations for a conception of colorful creatures, whereas without color connotations the conceived creatures would be colorless.*

Technically, for every set of connotations, a perception of all the possible subsets of connotations is first defined. This provides the maximal 'conceived environment':

Definition 3 *For a set of connotations $\mathcal{I}$, the* Universal Conceived Perception with $\mathcal{I}$ *is* $\mathcal{U}_{\mathcal{I}} = \langle 2^{\mathcal{I}}, \mathcal{I}, \delta \rangle$. *It has a* conceived environment *that consists of the field of all subsets of $\mathcal{I}$, designated $2^{\mathcal{I}}$, and its p-predicate δ is such that for every connotation $\alpha \in \mathcal{I}$, and for every* w-element $\mathrm{w} \subset \mathcal{I}$, *$\delta(\mathrm{w}, \alpha) = t$ if and only if $\alpha \in \mathrm{w}$, otherwise $\delta(\mathrm{w}, \alpha) = f$.*

The universal conceived perception with $\mathcal{I}$ thus has a totally two valued p-predicate. For any subset of connotations it conceives of a w-element. Loosely: every subset of concepts has the potential of giving rise to a design. A perception that consists of n connotations could conceive of up to 2^n different w-elements. Exponents grow fast, and, hence, so do the possibilites of design.

The dual 'pro-analytic' construct, in the spirit of the duality discussed in section 2.4, is the universal perception of [5], where connotations are defined by subsets of w-elements. Loosely: every subset of examples has the potential of giving rise to a concept.

The possiblity to obtain new connotations from sets of examples and, dually, to conceive of new w-elements from sets of connotations, demonstrates the flexibility of the formalization to model dynamic perceptual change and to ground perception in experience. A set of constituents (connotations, w-elements) may be extended at any time (applying a suitable p-morphism) to include a new constituent that is defined by a suitable subset.

Remark 1 *Definition 3 and example 5 are just a technical start. The pre-theoretical intuition is that there should be more to a creative design than the creation of an assemblage of connotations. In particular, some of the invented creatures should, somehow, be 'more conceivable' than others. This will be systematized in a short while.*

For a given perception $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$, the exponentially large variety of possible environments lies between the empty environment and the universal conceived environment (of definition 3), that has $2^{|\mathcal{I}|}$ connotations. The actual environment $\mathcal{E}$ itself should, indeed, be one of them. In the general case, however, the actual environment is not precisely known, because the p-predicate ϱ is not totally two valued. Categorically, a 'total improvement' of ϱ would provide a total perception of $\mathcal{E}$:

Definition 4 *A p-morphism $h : \mathcal{P}_1 \to \mathcal{P}_2$ is an* improvement *if h is the identity both on $\mathcal{E}$ and on $\mathcal{I}$, but $\varrho_1 \neq \varrho_2$. If $\mathcal{P}_2$ happens to be totally two-valued, then h is a* total improvement.

By the no-blur condition on p-morphisms (from definition 2), perception is more defined, namely *improved*, in $\mathcal{P}_2$ of the last definition.

Example 6 *An improvement as in definition 4 may capture the perceptual change that occurs, for instance, when an animal is x-rayed so that more can be perceived: $\varrho_1(\mathrm{w}, \mathit{fractured\_limb}) = u$, but $\varrho_2(\mathrm{w}, \mathit{fractured\_limb}) \neq u$.*

The various possible improvements of a given perception are very much like various *possible worlds* for that perception [22].

3.1 Perceptions with Boolean Environments

A mathematical property of the field of all subsets (the *power set*) of $\mathcal{I}$, which is the conceived environment of $\mathcal{U}_{\mathcal{I}} = \langle 2^{\mathcal{I}}, \mathcal{I}, \delta \rangle$ from definition 3, is its closure under Boolean operations. A Boolean algebra is a mathematical object that is an algebraic system consisting of a set of elements, together with the binary Boolean operations $(\vee, \wedge)$ that obey the axioms of commutativity and distributivity. The top $\top$ and the bottom $\perp$ are the identity elements such that $x \wedge \top = x$ and $x \vee \perp = x$, and each element x has a complement, $\neg x$ such that $x \wedge \neg x = \perp$ and $x \vee \neg x = \top$. Boolean homomorphisms[1] are set mappings between Boolean algebras that preserve the operations $\vee, \wedge, \neg$. The two common examples of systems that are Boolean algebras are:

1. The algebra of sets, in which the operations are set union, set intersection, and set complementation. $2^{\mathcal{I}}$is such a Boolean algebra, where $\mathcal{I} = \top$ and $\emptyset = \perp$.
2. The algebra of propositions in which $\wedge$ stands for 'and', $\vee$ stands for 'or', and $\neg$ stands for 'not'. They are interpretable as logical formulas. The dominant view in AI is that the knowledge content of high level artificial mental processes ought to be represented by data structures with this property [17].

In the sequel Boolean homomorphisms will be applied to relate between the two paradigms.

One may obtain a union of subsets of connotations, an intersection of subsets of connotations, and a complement of a subset of connotations (namely the set $\neg A$ of all connotations that are not in a given set A). However, since subsets of $\mathcal{I}$ are w-elements in the conceived environment of $\mathcal{U}_{\mathcal{I}}$, then obtaining subsets of connotations from other subsets of connotations actually means conception of w-elements on the basis of other w-elements. Formalization of such a process opens the possiblity to systematize the idea of design using examples, similes, and metaphors. Loosely, one can specify a design by Boolean combinations of similes, stating that the design is conceived, for instance, by resemblance to '(example-A and example-B) or (example-C but unlike example-D)'...This is the idea behind Boolean combinations of w-elements that is proposed now. It will be applied to other perceptions and not only to $\mathcal{U}_{\mathcal{I}}$. The passage from operations on sets of connotations to combinations of examples (with 'and', 'or', 'not') is, in a sense, a transition from paradigm 1 to paradigm 2 above. It will be captured by a Boolean homomorphism.

The idea of environments that are closed under Boolean connectives could be puzzling: How should Boolean combinations of w-elements be perceived? Since there is no authentic reality here to experience and apprehend, perception becomes an abstract process and should be conducted with care. Imaginative design is, indeed, a trying cognitive process that calls upon an 'inner eye'. In the artificial context it is now offered to guide the 'inner eye' with formal tools.

In the basic definition 1 of perceptions, every w-element could be perceived as having any connotation, without regard to perception of other w-elements. If the set of w-elements is closed under Boolean connectives, then the Boolean structure calls for some kind of dependence between the p-predicate values of different w-elements that are dependent in the Boolean sense. As an example, if two w-elements x, y have the connotation α, then conceiving of $x \wedge y$ (i.e. a design based on x and y) as lacking α 'does not seem right'. This intuition coincides

[1] This is a categorical construct in the spirit of section 2.5.

with the formal tools, since, technically, this would disgrace the qualification of ϱ as a *predicate* defined on a Boolean algebra.

In the case of a total, two valued, p-predicate there is a known classical schema which serves to define a two valued predicate on a Boolean algebra. (It is based on a Boolean homomorphism from the Boolean algebra under consideration to the two-element Boolean algebra $\{t, f\}$.) In the present case, however, the p-predicate is three-valued. One needs to find a sensible way to embed a three-valued predicate in a Boolean algebra.

The solution is in the abstraction of w-elements by subsets of connotations, as in the universal conceived perception $\mathcal{U}_\mathcal{I} = \langle 2^\mathcal{I}, \mathcal{I}, \delta \rangle$. It is easy to see that the p-predicate δ of $\mathcal{U}_\mathcal{I}$ is 'well behaved': If, for example, both w-elements $x, y \subset \mathcal{I}$ have a connotation α, then this connotation is an element in both subsets x and y. α is thus an element of both their union $x \cup y$ and their intersection $x \cap y$, and hence the w-elements $x \vee y$ as well as $x \wedge y$ have the connotation α, as expected. Also, the negated w-elements $\neg x$ and $\neg y$ do not have the connotation α because they consist of the respective complement sets.

Abstraction of w-elements as subsets of connotations is a cognitive process, and the tools that have just been devised for that purpose are p-morphisms. In this case p-morphisms into the universal conceived perception are applied. P-morphisms from any perception $\mathcal{P}$ into $\mathcal{U}_\mathcal{I}$ always exist:

- Let $\widehat{h} : \mathcal{P} \to \widehat{\mathcal{P}}$ be a total improvement of $\mathcal{P}$. (See definition 4).
- Define the natural p-morphism η from $\widehat{\mathcal{P}}$ into $\mathcal{U}_\mathcal{I}$ by the identity on connotations and the following mapping of w-elements:
  $\eta(\mathrm{w}) = \{\alpha \in \mathcal{I} \| \widehat{\varrho}(w, \alpha) = t\}$.
- $h = \widehat{h} \circ \eta$ defines a p-morphism from $\mathcal{P}$ to $\mathcal{U}_\mathcal{I}$.

The definition of h is just a schema: a possibly large number of p-morphisms is defined, with specific instances provided by different $\widehat{h}$ instances, since undefined p-predicate values could be unblurred either to t or to f. For $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$ with a Boolean algebra of w-elements to be an adequate perception of a Boolean environment, it can be formally shown to be both necessary and sufficient that at least one of the instances of h should be based on a mapping that is a Boolean homomorphism. At this point the duality principle of artificial perceptions from section 2.4 may be applied: A 'twin' Boolean construction was already deduced for sets of connotations that are closed under Boolean operations, for purposes of mental representations. Technical effort can be spared. (An insight into the connaturality of cognitive processes is also gained, to be discussed in a short while.) Only the results are quoted below, and readers interested in mathematical detail are refered to [4, p. 207–212]).

Theorem 1 *Let $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$ be a perception with a Boolean environment. The Truth Tables for the Boolean p-predicate ϱ are given in tables 1, 2, 3.*

Remark 2 *In tables 1 and 2, $\leq$ designates the Boolean partial order. In addition to the description from the beginning of section 3.1, a Boolean algebra can be alternatively described as a complemented distributive lattice. The lattice property of Boolean algera is that $x \vee y$ is the smallest element such that*

Table 1: Disjunction of w-elements:$\varrho(x \vee y, \alpha)$

$\varrho(y,\alpha)$ / $\varrho(x,\alpha)$	t	f	u
t	t	t	t
f	t	f	u
u	t	u	t if $(\neg x) \leq y$ (also $(\neg y) \leq x$ and $x \vee y = \top$); u otherwise

Table 2: Conjunction of w-elements:$\varrho(x \wedge y, \alpha)$

$\varrho(y,\alpha)$ / $\varrho(x,\alpha)$	t	f	u
t	t	f	u
f	f	f	f
u	u	f	f if $x \leq (\neg y)$ (also $y \leq (\neg x)$ and $x \wedge y = \bot$); u otherwise

$x, y \leq x \vee y$, and $x \wedge y$ is the greatest element such that $x \wedge y \leq x, y$. The partial order of the lattice is defined by the Law of Consistency*:*

$$\underline{x \leq y} \text{ if and only if } \underline{x \wedge \neg y = \bot} \text{ if and only if } \underline{\neg x \vee y = \top}$$

For w-elements x and y, $x \leq y$ describes the delineation of w-elements, namely that x lies below *y. This is why $\bot$ is called 'bottom', and $\top$ is called 'top'. (Set inclusion $\subseteq$ in the algebra of sets is a typical case of a Boolean partial order: a subset lies below its superset.)*

Loosely, the import of the tables is that disjunction of two w-elements is conceived as another w-element that should have the union of the connotations that they have, and should lack the intersection of the connotations that they lack. Conjunction of w-elements is dual: it is conceived as another w-element that should have the intersection of the connotations that they have, and should lack the union of the connotations that they lack. A negated w-element is a w-element that has the connotations that the original w-element lacks, and lacks the connotations that the original w-element has. Negated w-elements do seem less intuitive because the inversion of truth values alienates the simile. However, even in the human context negative examples are acceptable in many domains, and are often considered better than no example at all (e.g. 'This is exactly what you *do not* want to do'...).

Table 3: Negation of w-elements

$\varrho(\mathrm{w},\alpha)$	$\varrho(\neg\mathrm{w},\alpha)$
t	f
f	t
u	u

It is noted that the truth tables are not an arbitrary choice of some three-valued logic, but rather a result of the global categorical construction, based on the abstraction of w-elements as subsets of connotations. Theorem 1 provides a deductive apparatus that may be algorithmically applied for the computation of specific values of a p-predicate of a perception with a Boolean environment. These truth tables show that the categorical definition yields a p-predicate that is 'Boolean adequate' in a certain common sense of the term.

Remark 3 *A union of sets of connotations might try to patch together connotations that are mutually exclusive, such as, for example,* 'square' *and* 'round'*. In that case the w-element that is supposed to have these connotations is inconceivable. A perception with a complete mental representation, as described in [4], should be able to 'sense' the problem, because its representation records the fact, based on perceptual observations, that* 'square' *implies 'not* round'*, and* 'round' *implies 'not* square'*.*

A creative designing intelligence could sometimes modify the Boolean expression to obtain a conceivable design applying one of several options:

- *Change the specification to indicate preferal of one connotation over the contradicting one.*
- *Remove contradicting connotations from the specification, essentially indicating that these connotations are irrelevant to the design. Namely perceived as undefined for that design.*
- *Replace the coincidence of the contradicting connotations by their 'exclusive or', essentially indicating that exactly one of them should hold. This can be effected in the context of perceptions with Boolean algebras of connotations as studied in [4].*
- *Creatively replace the coincidence of the contradicting connotations by yet another connotation that captures a middle or a compromise, such as, for example, designing a* 'square with rounded corners' *instead of the inconceivable coincidence of* 'square' *and* 'round'*.*

Transitions to the modified specifications can be effected with suitable categorical constructs that are based on p-morphisms. This issue will be touched upon once more in section 4.2 below.

3.2 Categorical Context for Boolean Environments

(Readers not interested in mathematical context may skip this subsection.)

Technically, the *Subcategory of Perceptions with Boolean Environments* is defined, where perceptions are such that:

- Environments are closed under Boolean operations: they include the disjunctions, conjunctions, and complements of their w-elements.
- P-predicates conform to the truth tables of theorem 1.
- P-morphisms are such that, in terms of definition 2, $h : \mathcal{E}_1 \rightarrow \mathcal{E}_2$ is a Boolean homomorphism.

P-morphisms that map w-elements capture generalized analogies between environments, and these transitions are studied in [2]. Analogies in the subcategory of perceptions with Boolean environments capture what cognitive scientists call 'structural alignment' of the Boolean structure. This provides an insight into a connection between design processes and analogy making. This issue will be pursued in a short while.

4 Easily Conceived, Realistic Designs

In this section a formalized cognitive process is proposed that replaces abstract conception of subsets of connotations by reference to the actual environment, so that elements of this environment ought to provide similes, examples, and metaphors to assist the challenging process of design.

Design by conception of collections of connotations could often be too general and abstract. The challenge appears to be in that there is no immediate 'reality' for the sensory-motor-neural apparatus to experience and apprehend. As argued in the introduction, there is often more to a creative conception of a cohesive whole than merely a structured 'hanging together' of pointlike constituents. It is probably due to the fact that 'The whole is more than the sum of its parts', that a creative synthesis of a new entity is often more trying than a critical analysis of an already existing entity. For a perceptive design of a cohesive whole to emerge properly out of a skeletal assemblage of constituents, it is proposed that a designing intelligence should invariably refer to the actual environment. The phenomena in this environment represent actual cohesive wholes that should ground internal conception of designs and their prospective realizations.

Example 7 *Consider evolution theorists that use their scientific perception to conceive of hypothetical extinct creatures in the evolutionary line, such as 'the missing link [2]'. This cognitive process is achieved by 'imagination' of creatures that are based, for example, on the conjunction of a chimpanzee and a human, namely conception of a creature that features the intersection of properties as in table 2. Scientists employ their scientific perceptions to apply ontological properties as connotations. The cognitive process is grounded in the actual environment of living animals, because the imaginary w-element is described in terms of actually existing w-elements (namely humans and chimpanzees), using the Boolean connectives. That is more concrete than conception of extensions of skeletal combinations of connotations, independent of the actual environment.*

The pre-theoretical intuition about the need to invariably refer to the actual environment meets the intuition that some w-elements should be more easily conceived than others. Both intuitions are captured when conceived w-elements are expressed as Boolean combinations of existing, cohesive, perceptible, examples.

The first distinction that this study offers between more conceivable and less conceivable w-elements is based on a formalization of the distinction between conception of a skeleton of connotations on one hand, and a conception of a combination of actual examples. The need for similes, examples and metaphors

[2] A hypothtical extinct creature halfway in the evolutionary line between modern humans and their anthropoid progenitors. The relationship of humans to the present anthropoid apes (e.g. chimpanzees) is through common, extinct, ancestors.

to assist conception of ideas dates back to early imaginative creations: *'And God said, Let us make Mankind in our image, after our likeness ... '* (Genesis 1, 26). It is suggestive that a single Hebrew word has always designated both 'imagination' and 'likeness' (also 'resemblance' and 'similarity').

4.1 Free Generation of Conceived Boolean Environments

Based on the last considerations, one would like to formalize cognitive transitions from basic neural-sensory-motor perceptions into perceptions with conceived environments that are Boolean closures of the actual environment, generating perceptions with 'Vorstellung'. Namely, one needs to:

- Formally define the perception with a Boolean environment that consists of, *and only of*, w-elements that can be described as Boolean combinations of w-elements from the actual environment.
- Formalize the cognitive transition to that perception.

The setting where designs of new w-elements are conceived in terms of authentic examples that are there to actually perceive:

1. Should provide a way to refer to the actual environment while conceiving of that w-element in a concrete, simplified, manner.
2. Could suggest a possible plan for the actual design of that w-element, a plan that 'bottoms out' at familiar environmental entities. If z is conceived as a Boolean combination of x and y, then one may consider actually 'producing' z from (replications of) x and y.

Following former examples, scientists indeed produce new species through hybridization of existing ones, and resourceful crafts people design things by a perceptive use of *available* materials. In the kitchen, creativity is often based on the adaptation of a familiar receipe, and such variations are typically easier than starting 'from scratch'.

The technical 'twin' process (namely generation of a Boolean closure) was already shown for Boolean closures of sets of connotations, in the context of mental representation formation. Again, the duality principle for artificial perceptions can be applied. The consruction is easily adapted to Boolean closures of environments. The main ideas, constructions, and results are roughly sketched below. Readers interested in more mathematical detail are refered to [4, p. 212–217].

- The simplest and most general way to close $\mathcal{E}$ under Boolean operations is to take this set of authenic w-elements as *free generators*. The free Boolean algebra over $\mathcal{E}$ is designated ${\mathcal{B}_{\mathcal{E}}}^{\text{free}}$. Hence, the conceived Boolean environment is settled.
- The internal representation, namely the connotation set $\mathcal{I}$, is unchanged, so it remains to define the p-predicate ϱ^{free} for the Boolean closure ${\mathcal{B}_{\mathcal{E}}}^{\text{free}}$. The expected way to do this is to use the truth tables of theorem 1. The values of the p-predicate are immediate for w-elements in the actual, generating, environment. Since $\mathcal{E}$ is a set of generators for ${\mathcal{B}_{\mathcal{E}}}^{\text{free}}$, the computation for their Boolean combinations will eventually 'bottom out'

at these w-elements of the generating environment. The truth tables thus provide a deductive apparatus which may guide the computation of the values of the p-predicate for the entire Boolean environment from the generating perception.

- In the spirit of the proposed categorical formalization, the cognitive transition to the perception of the free Boolean closure of $\mathcal{E}$ is captured by a p-morphism:
$$\xi^{\text{free}} : \langle \mathcal{E}, \mathcal{I}, \varrho \rangle \to \langle \mathcal{B}_{\mathcal{E}}{}^{\text{free}}, \mathcal{I}, \varrho^{\text{free}} \rangle$$
The underlying mapping ξ^{free} is simply the inclusion map of generators. Loosely, the existing environment is naturally embedded into the imagined environment that extends it.

The formalized cognitive process that is described above is categorical in the sense that it provides tools to model particular cognitive transitions of imagination for particular perceptions of particular environments, and at the same time it is sufficient as a general account of all such transitions. In categorical terminology, a *free functor* from the category of perceptions into the subcategory of perceptions with a Boolean environment has just been defined:

$$\mathcal{G}^{\text{free}} \ : \ \mathcal{P}rc \to \mathcal{P}rc^{\text{bl}-\mathcal{E}}$$

and the exact mapping of constituents (w-elements) is defined by the *natural transformation* $\xi^{\text{free}} : \mathcal{P} \to \mathcal{P}^{\text{free}-\text{bl}}$. The categorical terminology provides a precise and testable form of stating that the process is open-minded, consistent, and methodical and that it preserves analogies between environments:

- The freedom of the construction warrants the broadest, unrestricted, open-minded, view, in that any Boolean combination of w-elements and any improvement (see definition 4) of the generating perception is taken into account and none are overlooked.

- If there exists an analogy (i.e. a p-morphism) between perceptions of two environments, then (an extension of) the same analogy will hold between the Boolean environments that are generated: perceptions of analogical environments will generate perceptions of similarily analogical Boolean environments.

- The extended analogy will feature structural alignment of the Boolean structure (i.e. is based on a Boolean homomorphism between the Boolean environments), meaning that analogical actual environments will generate conceived environments that are similarily structured: there is consistent method in the process.

- The same analogy that is mentioned above holds whether one performs the analogical transition on the actual, generating, environments, or delays the analogical transition until after the Boolean environment is generated.

The expressive power of the mathematical formalism is in grasping the itemized, complex yet loose, verbal description above in one commutative diagram (figure 1). This is an instance where categorization of perceptions provides us with tools of scrutiny that capture pre-theoretical intuitions about cognitive perceptions.

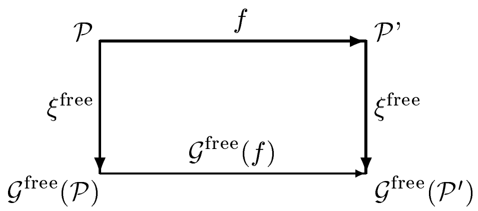

Figure 1: Conceived Boolean Environment Generation with Analogies

As mentioned before, formalization of mental representation formation followed the dual mathematical construction for sets of connotations (instead of sets of w-elements). The properties of the categorical construction warranted preservation of interpretive transitions between perceptions and ontologies, in the same manner as the present construction has just warranted preservation of analogies. It follows that the duality principle of artificial perceptions has not only saved us quite a few pages of mathematical carpentry, but has also provided us with an insight: imaginative transitions to conceived environments are connatural to the cognitive formation of a mental representation. The former is 'pro-synthetic' in that it takes cohesive, existing, w-elements as its basic building blocks, while the latter is 'pro-analytic' in that it takes simple, basic, sense connotations as its building blocks. The required computational capabilities are the same.

In the context of free generation of perceptions with Boolean environments, there is a clear distinction between *existing* and *imagined* w-elements. Existing w-elements are generators of the Boolean algebra of w-elements. They are actually perceived in the generating $\mathcal{E}$. Imagined w-elements are internally conceived, and their perception typically involves computation on top of simple sensory-motor-neural perception. *In the proposed formalization computation and complex constructs capture mental effort.* The complexity of the Boolean expression that describes a design provides basis for testable measures of

1. The extent of the 'mental effort' that is required to conceive the design: more complex Boolean expressions stand for more mental effort.

2. The feasibility of the plan for the actual design of the w-element that is suggested by the Boolean expression, and the prospect that the actual effect of the design will emerge successfully as a cohesive whole: More complex Boolean expressions stand for more complex plans and, consequently, more pitfalls along the way.

The two items above correspond to the two enumerated items from the beginning of this section.

The second distinction that this study offers between more conceivable and less conceivable w-elements is based on the complexity of the Boolean expression as just explained. The derivative of the measures depends, of course, on the basic perception and environment that generate the process. Some environments and their w-elements are, indeed, closer to what one might fancy than others. As an example, it is probably easier to conceive of a detailed imaginary marsupial (pouch) mammal when in an Australian zoo than elsewhere, because actual perception of that environment provides authentic, cohesive, wholes that are closer to the final design (namely the Boolean expression that generates the design should be less complex than elsewhere).

4.2 Hybrid Designs

In remark 3 of section 3.1 it was mentioned that a creative designing intelligence might sometimes need to modify a Boolean expression to improve a design that is based just on limited combinations from a given Boolean environment. This might call for a 'hybrid' approach: the designing artifact perceives the Boolean closure of w-elements as defined in this section, conceiving of designs in terms of examples and similes from the actual environment. In the absence of a Boolean combination of w-elements that satisfies its exact needs, it might apply a combination of examples that is close to what it is looking for, then fiddle with additional modifications related to 'singular' connotations that the combination of examples cannot handle. (namely additional Boolean combinations with subsets consisting of these singular connotations). This is a 'hybrid' approach because it applies examples and similes as suggested in this section, with an additional abstract manoeuver of subsets of connotations as described in section 3.

In the proposed framework this implies extension of the Boolean environment, using a suitable p-morphism, to include abstract w-elements that are subsets of connotations as described in section 3, then conception of a Boolean combination in the extended environment. This means greater mental effort:

- An additional extension (p-morphism) of the conceived environment is applied.
- The Boolean expression that describes the design becomes more complex.
- Although the basic design is based on actual examples, the final 'polish' is rather based on abstract manoeuver of (subsets of) connotations.

The reward for the mental effort should be a successful design. (It is expected that the complexity of a solution should be proportional to the complexity of the problem.)

5 Perceptual Acuity and Creative Solutions

In this section a refinement to the Boolean generation of the former section is proposed. Observation of relations among the constituents of a design is formalized, and these observations are incorporated into the conceived Boolean structure.

There are often restrictions on the manner in which entities can be composed out of their constituents. Intelligence is typically marked by a discerning perception and understanding of its environment, and a further pre-theoretical intuition is that observation and internalization of environmental patterns should also be performed by an intelligent perceptive artifact. Design processes ought to benefit from these capabilities.

Philosophers of science [8] argue that every rational generalization may be typically analyzed as 'everything that is A is B', provided that A and B are sufficiently complex properties [3]. The formal tools for capturing observation of lawlike patterns in $\mathcal{E}$, based on a given perception $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$, are:

Definition 5 *Let $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$ be a perception. A w-element $x \in \mathcal{E}$ is* subjacent *to another w-element $y \in \mathcal{E}$, designated $x \trianglelefteq y$, if, for all connotations α in $\mathcal{I}$ the two following conditions hold:*

- $\varrho(x, \alpha) = t$ *implies that* $\varrho(y, \alpha) = t$.
- $\varrho(y, \alpha) = f$ *implies that* $\varrho(x, \alpha) = f$.

Definition 6 *Let $\mathcal{P} = \langle \mathcal{E}, \mathcal{I}, \varrho \rangle$ be a perception. Two w-elements x, y in $\mathcal{E}$ are* congeneric *if, for all connotations α in $\mathcal{I}$, $\varrho(x, \alpha) = \varrho(y, \alpha)$.*

The congeneric relation is an equivalence relation (actually, it is a two-sided subjacency). The subjacency relation is a quasi ordering on $\mathcal{E}$, and a partial ordering on the equivalence classes of the congeneric relation. These relations can also be defined between Boolean combinations of w-elements, such as the imagined w-elements in the conceived environments of section 4.

Example 8 *Returning to example 7, the conjunction of w-elements* human $\wedge$ chimpanzee *is subjacent to each one of the w-elements* human, chimpanzee. *This can be easily verified from truth table 2 and from the last definitions.*

The congeneric and subjacency relations depend, indeed, on the classifications implied by the connotations of that perception. When evolution theorists define hierarchies of species, they use the primitives of their scientific classifications for that cognitive process in a similar way. Non-scientific perceptions would typically imply different relations. A florist would maybe connote flowers by their color, size, and price. The florist would hence classify as congeneric flowers of the same color, size, and price. This is a classification that is foreign to the botanist's classification, being applied by a perception with different goals. In a similar sense, various newspapers are more likely to be congeneric as papier-mâché raw material than in the case where perception refers to their printed contents. All perceptions are legitimate, each serving own objectives.

Creative design is related to perception of lawlike patterns in a variety of ways:

- Lawlike patterns in the environment often have to be preserved by design processes, and hence they restrict the possibilities of design (e.g. all bridges have supports, bridge$\trianglelefteq$support, therefore one should not conceive of bridges without supports).

  When a designing intelligence restricts design processes according to perceived lawlike patterns, there is often, though not always, an implicit assumption about the causal nature of the relation, namely that there is a reason behind that pattern that is necessary for the effectiveness of the design (e.g. the bridge will collapse without proper support).

- Contrary to the former item, creative breakthroughs are often achieved by disrupting a 'mental set' on presuppositions. Patterns should not be internalized to a point where one's open-minded, free, perception is 'blinded'

[3] Boolean operations have been proposed here as the priciples for setting up A and B from simple perceptual constituents.

by them. Some patterns could, sometimes, be disrupted or replaced by other patterns in a way that actually contributes to the effectiveness of the design. For example, old world arch support for bridges is often replaced by suspension support in modern bridges. The pattern $bridge \trianglelefteq arch$ was creatively disrupted.

When a designing intelligence disrupts or replaces a pattern, it is often, though not always, the result of a 'dialogue' between a creative perception and its reasoning high-level processes regarding the causal nature of the pattern. The issue is whether there is a reason behind the pattern that is necessary for the effectiveness of the design, or maybe it could be disrupted (or replaced by another pattern) in a way that does not impair the design, and may even contribute to its effectiveness.

- Creative originality is often related to the generation of *discrepancies*. Differences from the norm could be fascinating [4], and surrealistic art is just one example. However, a designing artifact should typically observe the contingency before disrupting it. A creative discrepancy is salient against a 'background' of contingencies. When everything is radically disrupted and all basic patterns are lost creativity often misses its point.

- Perceptive creativity is sometimes related to enhanced or exaggerated patterns, as in caricatures.

- 'Style' is sometimes related to specialized, additional, lawlike patterns that designing intelligences develop or recreate, producing own 'signatures', 'local color', or a certain 'atmosphere'. For instance, to specially conjure an *old world* connotation, a designing intelligence might choose to preserve the pattern $bridge \trianglelefteq arch$.

Subjacencies are not meant to express universal facts, they are environment and perception specific. Also, observation of a lawlike pattern has nothing to do with an explanation of *why* the pattern holds [5]. The observation may, indeed, stimulate reasoning high-level processes to explain the causes of the pattern, but this is a separate cognitive process. It was mentioned above that a 'dialogue' between a creative perception and its analytic high-level processes could be applied to untie the tangled knot between, on one hand, some good reasons to preserve patterns and, on the other hand, other good reasons to disrupt them.

A fundamental necessity that all perceptions with Boolean environments feature is, at least, that the Boolean structure never introduces patterns that are not observable. In mathematical terms, a perception with a Boolean environment $\mathcal{E}$ is always *valid* in the sense that the Boolean partial order in the Boolean algebra $\mathcal{E}$ is a subset of the subjacency relation. If $x \leq y$ in the Boolean algebra (see remark 2), then the perceptual subjacency $x \trianglelefteq y$ holds as well (see example 8). This minimal requirement is featured by all Boolean constructs. (The relevant theorem is proven for mental representations in [4, p.218].)

[4] In the human context, the need to depart from the norm for purposes of creative design is sometimes applied to explain the relatively high number of creative people who were unstable: Gustav Mahler, Robert Schumann, Vincent Van Gogh, Friedrich Nietzsche, Franz Schubert, Nikolay Gogol, Fyodor Dostoyevsky...

[5] Perception follows, in this context, the scientific enquiry principle *Hypotheses non fingo* [6, p.261(Newton)].

However, one might sometimes expect more than that: When an acute perception performs a natural transformation (as in section 4) to a Boolean environment, it could be expected to incorporate into the boolean structure some observations of lawlike patterns. By doing this, observed subjacencies between w-elements are introduced into the conception of designs, so that intelligent design could eventually benefit from a discerning perception and understanding of its environment.

Example 9 *Following former examples, what if evolution theorists had generated a 'free hierarchy of species', without perceiving that some existing species are, indeed, related to other existing species? One could, indeed, expect of a discerning scientist to apply observations to the classification in such a way that the relations between existing species should be delineated by scientific ontologies. Namely, if x precedes y in the evolutionary line, then it should be that x* is below *y in the hierarchy as well (also when either x, or y, or both, are hypothetical, conceived combinations of other w-elements).*

More perceptual acuity, as just described, is captured whenever a perception with a Boolean environment is also *complete*. Formally: for any two w-elements x, y in a perception with a Boolean environment, if, perceptually, $x \trianglelefteq y$, it should also be true that $x \leq y$ in the Boolean algebra.

The second canonical natural transformation, the one that is proposed now, differs from the natural transformation of section 4 in perceptual acuity as just explained. While the former construction is technically based on a general free construct, the present construction is more 'perceptually acute'. Its Boolean construct delineates the full subjacency relation (one may also choose to sketch only part of it, creating original discrepancies). It is technically captured by another functor.

As anticipated before, whenever perceptions take commitments to patterns there is a price in open-mindedness. If a Boolean environment delineates subjacencies and congenerics, it will not support any cognitive process (e.g. an analogy) that violates them. All mappings and structural alignments now need to respect the patterns. Consider the free generation of section 4. The more perceptually acute generation that is considered now 'moves things around' in the Boolean structure. W-elements that were originally perceived independent of one another in the free structure are now dependent and placed 'one below the other'. It follows that Boolean mappings that can be extended to mappings between the new structures need to be restricted as well. They are designated *monotone* Boolean p-morphisms.

The main steps of the construction are roughly sketched below. Readers interested in mathematical detail are refered to [4, p. 220–230], where a technically similar schema is studied with respect to mental representation formation. A variant where only some of the patterns are preserved, allowing discrepancies, is discussed on (p. 233–234).

- The basis for the Boolean construct is, again, the free Boolean algebra $\mathcal{B}_{\mathcal{E}}{}^{\text{free}}$ as in the free generation of section 4. A delineation of subjacencies is effected by taking the algebra modulu a suitable ideal (loosely: the ideal generated by elements of the form $x \wedge \neg y$, where $x \trianglelefteq y$ is a subjacency), yielding a more constrained Boolean algebra, $\mathcal{B}_{\mathcal{E}}{}^{\text{complete}}$. As a result, the authentic w-elements of $\mathcal{E}$ are still generators of the Boolean algebra, but not necessarily *free* generators. There may now be Boolean dependencies among them (of the form $x \leq y$), capturing internalization of perceptual observations.

- The p-predicate $\varrho^{\text{complete}}$ for the modified Boolean closure is computed, as for all perceptions with Boolean environments, according to the truth tables of theorem 1. *The change is in the Boolean partial order* of the modified Boolean closure, which effects computation of the p-predicate as specified in the lower right entries of the conjunction and the disjunction tables. As before, since $\mathcal{E}$ is a set of generators for $\mathcal{B}_{\mathcal{E}}{}^{\text{complete}}$, the computation will eventually 'bottom out' at the w-elements of the generating environment.

- In the spirit of the proposed theory, the cognitive transition to the perception of this perceptually acute Boolean closure of $\mathcal{E}$ is captured by a p-morphism: $\xi^{\text{complete}} : \langle \mathcal{E}, \mathcal{I}, \varrho \rangle \to \langle \mathcal{B}_{\mathcal{E}}{}^{\text{complete}}, \mathcal{I}, \varrho^{\text{complete}} \rangle$, and the underlying mapping ξ^{complete} is simply the inclusion map of generators: authentic w-elements are naturally embedded into the extended, conceived environment.

The new construct is now compared with the free genration of section 4:

- As in the free generation, all Boolean combinations of w-elements are conceived.

- Unlike free generation, improvements (see definition 4) of the generating perception are expected to agree with the delineation of patterns. For example, if an animal is considered to be a descendant of another animal in the evolutionary line, then any new fact that will be learned about the ancestor is automatically assumed to hold for the descendant.

- As in the free generation, analogies (i.e. p-morphisms) that hold between simple perceptions can be extended to analogies between the new Boolean closures. However, unlike free generation, this is true only if their extensions to analogies between the free closures are monotone, namely only if there is no violation of patterns that were introduced into the new Boolean closure. In that case the extended analogies will feature structural alignment of both the free Boolean structure and perceptually observed patterns. It follows that acute perceptions of analogical actual environments will generate conceived environments that are similarily structured: there is consistent method in the process.

- In the case of acute perceptions and structured analogies as above, the same analogy holds whether one performs the analogical transition on the actual, generating, environments, or delays the analogical transition until after the Boolean environment is generated.

In categorical terminology, a *free functor* has just been defined, from the category of perceptions *with structured analogies* into the subcategory of perceptions with Boolean environments:

$$\mathcal{G}^{\text{complete}} : \mathcal{P}rc^{\text{strct}-\mathcal{E}} \to \mathcal{P}rc^{\text{bl}-\mathcal{E}}$$

and the exact mapping of cognitive constituents (w-elements) is defined by the *natural transformation* $\xi^{\text{complete}} : \mathcal{P} \to \mathcal{P}^{\text{complete-bl}}$.

Once again, the categorical terminology provides a precise and testable form of stating that the process is committed to the chosen lawlike patterns, *and only to them*, consistent, and methodical in the structure that it incorporates into the conceived environment, and in preserving analogies between environments. A commutative diagram similar to the one in figure 1 (with 'complete' replacing 'free'), provides a powerful and rigorous expression of the properties that are loosely itemized in the comparison above. Categorization of perceptions provided us, once more, with tools of scrutiny that capture pre-theoretical intuitions about cognitive perceptions.

As mentioned before, formalization of perceptually acute mental representation formation followed the dual mathematical construction for sets of connotations (instead of sets of w-elements). The functor that generates a complete perception of a Boolean environment over any basic perception is a technical twin to the one that generated a complete Boolean mental representation. Again, the duality principle of artificial perceptions from 2.4 is applied, based on the technical symmetry between w-elements and connotations. In forming mental representations, an acute perception is expected to supplement the general logical structure of connotations with perceptually based observations of 'synonym' connotations (the twin construct to congeneric w-elements) and connotations that subsume one another (the twin construct to subjacencies between w-elements). This is connatural to the process that is presented here: supplementing the Boolean environment with perceptually based observations of subjacencies congenerics. The properties of the categorical construction warranted preservation of structured, perceptually acute interpretive transitions between perceptions and ontologies, in the same manner as the present construction has just warranted preservation of structured, perceptually acute, analogies. Again, the duality principle of artificial perceptions has not only saved us quite a few pages of mathematical carpentry, but has also provided us with an insight: Acute perceptions could employ their capabilities in the same manner for imaginative transitions to conceived environments and for mental representations. The mathematical categorical construct provides a unified, context-free, methodology for these connatural cognitive transitions.

One essential difference between the free generation of perceptions with Boolean environments of section 4, and the more structured generation of this section, is that now there is no more clear-cut distinction between existing w-elements and, on the other hand, imagined w-elements.

Example 10 *In an animal environment, it is not necessary to make the effort to mentally conceive a hybrid between a horse and an ass when one can actually examine a mule. 'mule' is a w-element that should be now found at the Boolean node for* (ass $\vee$ horse)*. In the general construction of section 4, the w-element* mule *would have been a free generator that is independent of other existing w-elements, 'horse' and 'ass' in particular. In the more structured, perceptually acute construction that is offered now,* mule=horse$\vee$ass*, resulting in a different Boolean structure.*

The natural transformation that has just been described thus captures a cognitive transition where perception 'realizes' that some existing w-element is actually what a Boolean combination conceives. An existing w-element is congeneric to a non-trivial Boolean combination of other existing w-elements, and the construction warrants that it is merged with that combination, capturing a cognitive internalization of that fact. There may also be cases where a rather complex Boolean combination of w-elements gets merged with a somewhat less complex Boolean combination of w-elements. Practical implications are:

- Cognitive perception provides an authentic, cohesive whole to explore.
- Effusion of imaginative efforts and creative resources can be redirected to where they are genuinely required, instead of wasting them on something that already exists. This would be a case of a practical application of Ockham's razor principle, that 'entities should not be complicated beyond necessity' as acute perceptions provide simpler solutions to creative tasks.

The distinction that this study has offered between more conceivable and less conceivable w-elements, and was based on the complexity of Boolean expressions, is thus enhanced by the last construction.

6 Connatural Cognitive Processes

In section 2.4 the symmetry between the 'pro-analytic' and the 'pro-synthetic' aspects of artificial Perceptions was presented. From a purely technical, context free, point of view, the roles that a w-element and a connotation play in the definition of the p-predicate are interchangeable. 'Pro-synthetic', holistic, cognitive processes (such as analogy making and design) manoeuvre cohesive wholes as basic constituents. 'Pro-analytic' cognitive processses (such as interpretations and mental representation processes) break impressions of cohesive wholes into separate sense connotations and manoeuvre them as basic constituents.

In the constructions that were presented in this work this symmetry was repeatedly applied:

- Technically, formal constructions and theorems that had been established for connotations were applied to w-elements, mutatis mutandis, saving some tedious mathematical carpentry.
- Since the proposed formal constructions and theorems capture cognitive processes, the technical symmetry provided insights into similarities of cognitive processes and capabilities.

The import of the duality of artificial perceptions and related cognitive processes is summarized in table 4. An artificial perceptive artifact that performs any one of the cognitive processes in the table should be able to perform the technically dual process in the same row, since the same formal construct (i.e. implementing modul) ought to be reusable for both perceptual-cognitive processes. Generally speaking, design is connatural to reasoning.

In a human context, one could maybe say that researchers and scholars are typically adept at the processes in the 'pro-analytic' column, while artists and craftspeople typically master the processes in the 'pro-synthetic' column. A pro-analytic view enjoys the advantage of insights and understanding of fundamental properties, while the pro-synthetic view enjoys the advantage of holistic experience of 'wholes that are more than the sum of their parts'.

Table 4: Formally Connatural Cognitive Constructs and Processes

Internal Representation, Analysis and co.	**External Environment, Design, Synthesis and co.**	**Shared Formal Construct**
Basic sense connotations	Actual World Elements	Primitive (In the context of high-level perception)
Interpretations,	Literal Analogies	P-Morphisms
Examples give rise to Concepts	Concepts give rise to designs	Universal Construction with Power Sets
Complex Connotations	Conceived W-Elements	Boolean Expressions
Mental Representation Formation	Creative Conception of Extended Environments	Free Functor to a Boolean Subcategory
Synonym Connotations	Congeneric W-Elements	Equivalence Relation
Subsumed Connotations	Subjacent W-Elements	Quasi-order
Structured Interpretations (with synonyms and subsumptions)	Structured Analogies (with congenerics and subjacencies)	Pattern Preserving P-Morphisms
Perceptually Acute Boolean Representation Formation (with synonyms and subsumptions)	Perceptually Acute Creative Conception of Environments (with subgenerics and subjacencies)	Free Functor from the Pattern-Preserving Subcategory to a Boolean Subcategory

The processes and capailities in the last four rows of the table seem to depend on a single component that should be capable of a certain mental acuity: the detection of lawlike patterns of constituents (w-elements, connotations). That component should be reusable for various, seemingly different, intelligent perceptual cognitive processes. A programmed implementation of such a component is described in [7], where they describe LAD, a logic-based methodology for analyzing observations that is based on the detection of hidden patterns in the data. Combinations of such patterns are used for developing general classification procedures. It is proposed here that similar methodologies can be applied by an intelligent artifact to embodied ontologies, and that the classifications be used for more cognitive activities. With a neat distinction between subjective perceptions, external environments, and internal representations, the mathematical categorical toolkit of the theory of artificial perceptions provides basis for employing the Boolean methodology for a broader spectrum of cognitive activities.

7 Further Applications of Conceived Environments

The most obvious purpose of conceived w-elements is the purpose of design. Based on the formal tools that are introduced in this work an artificial agent should be able to internally conceive, evaluate, and devise a plan for the manipulation of its environment. However, internal conception of environmental entities is not always aimed at actually modifying the external environment. Example 7 presented a case where internal conception of entities (hypothetical extinct creatures in the evolutionary line) is performed for other purposes, scientific theorizing in that case.

Affective behavior is another domain where conceiving of things, just internally, could be applied. Emotions have been lately recognized as a necessary component of intelligent behavior that offers a rich potential for the design of artificial systems [28]. Emotions pervade intelligence at many levels. They are inseparable from cognition, perception, attention, memory, learning, decision making, social interaction and communication [18].

When emotive reactions are conjured in response to something in the environment (such as existential danger) they can be connected to perception. In [3] the theory of artificial perceptions is extended to couple basic connotations with primary emotive reactions that they might conjure. These 'instant' reactions are distinguished from rational reactions that are initiated by the higher level reasoning moduls, following a thoughtful consideration and careful analysis of all available aspects of the situation. Resource consuming rational behavior is not always an option in a system with bounded resources. Also, rationalization is not capable of arbitrating a reaction to all situations. Accepting the option that agents could react without solicitation of the reasoning modul captures a certain notion of artificial emotional capability. The Boolean constructions are employed to formalize complex affective behavior that emerges with the activation of multiple, possibly conflicting, emotive reactions (that might, in addition, produce conflicts with long term goals and rationalized behavior).

It turns out that if one wants artificial agents to participate in affective interaction with the environment and with other agents, more than immediate perception is required. Quite a few affective processes occur 'internally'. Agents need to predict their own emotive reactions and those of others when applying affective decision making, social interaction, or communication. That is where conceived environments come in. Conceiving of situations is a form of internal, 'mental', processe that could conjure emotive reactions, supplementing emotional capabilities in many ways. Human agents are familiar with the phenomenon of emotive reactions to mere thoughts or imagined situations. For example, a 'positive' emotive reaction could be conjured when a 'successful' design is internally conceived (anticipated success), and a 'negative' emotive reaction could be conjured when a 'failed' design is internally conceived (anticipated failure). Internally conceived perceptions of other affective agents enable an internal prediction of their reactions at a conceived situation. Empathy, which is at the roots of pro-social, moral and ethical behavior, is, in a sense, the result of an agent conceiving of itself in the situation of the other. Conceiving of hypothetical situations thus opens possibilities for a broad spectrum of emotional activity that is rich both in cognitive and in affective contents. Had this been a study of human processes, one could have said that this explains why artists are often considered to have enhanced emotional capabilities: they have a trained 'inner eye' and thereby they can apply a broad emotional repertoire.

8 Summary

The mathematical infrastructure that is provided by the definition and the study of the category of artificial perceptions has enabled us to extend, to systematize, and to formalize intuitive pre-theoretical conceptions about perceptive artificial transitions from perception of actual environments to an internal conception of other environments, providing the basis for creative design processes.

- The definitions started from perceptions of actual environments. W-elements in these environments are cohesive wholes that are susceptible to acute analysis by a perceiving intelligence.
- The formal construction started with a systematization of the intuitive idea that sets of connotations could give rise to designs.
- Whenever w-elements are abstracted as sets of connotations, one may obtain a union of sets, an intersection of sets, and a complement set. This formalizes conception of w-elements on the basis of other w-elements, and opens the possiblity to systematize the idea of grounding design using examples, similes, and metaphors. The formal construct devised for that purpose were perceptions with Boolean environments.
- Abstract design by assembly of connotations was then replaced by a free Generation of a conceived Boolean environment over any perception. This formalized cognitive process systematizes the grounding of design in an authenic environment. Examples from the actual environment provide cohesive wholes that ground the cognitive process and 'give airy nothing a local habitation and a name'.
- Observation of relations among possible examples that ground the design was formalized, and these observations were incorporated into the conceived Boolean structure, resulting in a refinement of the Boolean structure.
- Describing designs by means of Boolean expressions that involve examples from the immediate environment also provides:
  - A measure (by means of the complexity of the boolean expression) of the 'mental effort' that is required to conceive the design, as well as a measure for the prospect that the actual effect of the design will emerge successfully as a cohesive whole.
  - Suggestion of a plan for the actual design of the conceived w-element from (imitations of) existing w-elements, together with a measure (by means of the complexity of the boolean expression) of the complexity of the plan.
  - For a perceptually acute intelligent artifact, formal tools to spare effusion of creative resources by observing existing entities that can replace chunks of design.

By employing well-developed tools of mathematics, it is thus possible to capture complex processes of creative design, that could be grasped only loosely by verbal descriptions. The categorical framework models them in a precise, testable and applicable form.

It is often the case in AI and in cognitive science, and also in category theory and in logic, that one applies a methodology while studying it. This study itself constitutes a proposed theoretical design of an 'artificial design standard'. The constituents of the proposed design, the basis on which it is architectured and measured, are the primitives of the proposed category (w-element, connotation, perception predicate), supported by the categorical primitives and the Boolean primitives. The analysis and design in terms of a relatively small number of primitives predicts the possibility of tidily structured implementations with a reduced component set. It was also shown how context free 'mental' components (i.e. categorical and Boolean constructs) are reusable across a broad spectrum of artificial cognitive activity. Using the terminology of Magnan and Reyes [24], mathematical categorical constructs provide 'blueprints' for the design of cognitive activities. These 'blueprints' were applied here to design creative design.

References

[1] J.R. Anderson and C. Lebiere. *The Atomic Components of Thought.* Lawrence Erlbaum Associates, 1998.

[2] Z. Arzi-Gonczarowski. Percieve this as that – analogical cognitive transitions with categorical tools. In *Electronic Proceedings of the Fifth International Symposium on AI and Mathematics*, Fort Lauderdale, Florida, January 1998. http://rutcor.rutgers.edu/~amai.

[3] Z. Arzi-Gonczarowski. Wisely non rational – a categorical view of emotional cognitive artificial perceptions. In D. Cañamero, editor, *Papers from the 1998 AAAI Fall Symposium: Emotional and Intelligent – The Tangled Knot of Cognition*, pages 7–12, Orlando, Florida, October 1998. AAAI Press. Technical Report FS-98-03.

[4] Z. Arzi-Gonczarowski and D. Lehmann. From environments to representations–a mathematical theory of artificial perceptions. *Artificial Intelligence*, 102(2):187–247, July 1998.

[5] Z. Arzi-Gonczarowski and D. Lehmann. Introducing the mathematical category of artificial perceptions. *Annals of Mathematics and Artificial Intelligence*, 23(3,4):267–298, November 1998.

[6] S. Blackburn. *Dictionary of Philosophy.* Oxford University Press, 1996.

[7] E. Boros, P.L. Hammer, T. Ibaraki, A. Kogan, E. Mayoraz, and I. Muchnik. An implementation of logical analysis of data. Rutcor Research Report RRR 22-96, Rutgers University, New Brunswick, N.J., July 1996.

[8] R.B. Braithwaite. *Scientific Explanation.* 1953.

[9] M.A. Croon and F.J.R. Van de Vijver, editors. *Viability of Mathematical Models in the Social and Behavioral Sciences.* Swets and Zeitlinger B.V., Lisse, 1994.

[10] W.D. Ellis, editor. *A Source Book of Gestalt Psychology.* Routledge and Kegan Paul, London, 1938.

[11] B. Falkenhainer, K.D. Forbus, and D. Gentner. The structure–mapping engine. *Artificial Intelligence*, 41(1):1–63, 1990.

[12] G. Fauconnier. *Mappings in Thought and Language*. Cambridge University Press, 1997.

[13] R.M. French. *The Subtlety of Sameness–A Theory and Computer Model of Analogy Making*. A Bradford book, The MIT Press, Cambridge Massachusetts, London England, 1995.

[14] P. Gärdenfors. Induction, conceptual spaces and AI. *Philosophy of Science*, 57:78–95, 1990.

[15] P. Gärdenfors. Three levels of inductive inference. *Logic, Methodology, and Philosophy of Science*, IX:427–448, 1994.

[16] P. Gärdenfors. Meaning as conceptual structures. Technical Report 40, Lund Univesity Cognitive Studies, Lund, 1995.

[17] M. Genesereth and N.J. Nilsson. *Logical Foundations of Artificial Intelligence*. Morgan Publishers, San Mateo, CA, 1987.

[18] D. Goleman. *Emotional Intelligence*. Bantam Books, 1995.

[19] S. Harnad. The symbol grounding problem. *Physica*, D 42:335–346, 1990.

[20] D. Hofstadter. *Fluid Concepts and Creative Analogies - Computer Models of the Fundamental Mechanisms of Thought*. Basic Books, 1995.

[21] K.J. Holyoak and P. Thagard. *Mental Leaps – Analogy in Creative Thought*. A Bradford Book, the MIT Press, Cambridge, Massachusetts, 1995.

[22] S. Kripke. Semantical considerations on modal logic. *Acta Philosophica Fennica*, 16:83–94, 1963.

[23] F.W. Lawvere. Tools for the advancement of objective logic: Closed categories and toposes. In J. Macnamara and G.E. Reyes, editors, *The Logical Foundations of Cognition*, pages 43–55. Oxford University Press, 1994.

[24] F. Magnan and G.E. Reyes. Category theory as a conceptual tool in the study of cognition. In J. Macnamara and G.E. Reyes, editors, *The Logical Foundations of Cognition*, pages 57–90. Oxford University Press, 1994.

[25] M. Mitchell. *Analogy Mapping as Perception – A Computer Model*. A Bradford book, The MIT Press, Cambridge Massachusetts, London England, 1993.

[26] A. Newell. *Unified Theories of Cognition*. Harvard University Press, 1990.

[27] N.J. Nilsson. Eye on the prize. *AI Magazine*, 16(2):9–17, Summer 1995.

[28] R.W. Picard. *Affective Computing*. The MIT Press, 1997.

[29] S. Vosniadou and A. Ortony, editors. *Similarity and Analogical Reasoning*. Cambridge University Press, 1989.